STATISTICAL CHALLENGES IN 21ST CENTURY COSMOLOGY

Approximate Bayesian computation: an application to weak-lensing peak counts

Chieh-An Lin & Martin Kilbinger

SAp, CEA Saclay

Chania, Greece — May 26th, 2016

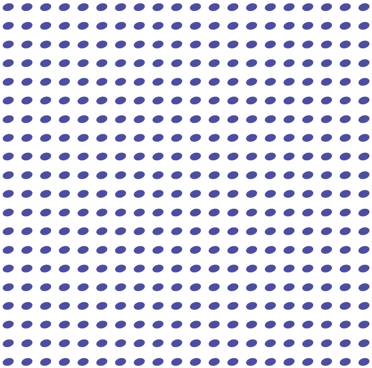
Outline

1. Weak-lensing peak counts

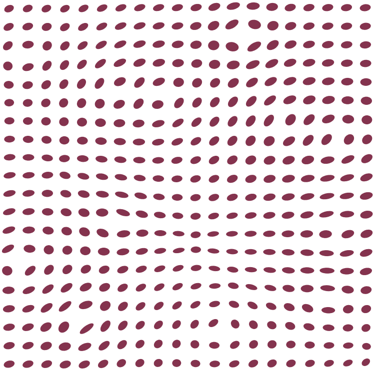
2. Approximate Bayesian computation

3. Application to survey data

Flashback on weak lensing

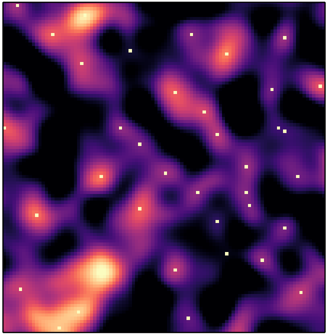


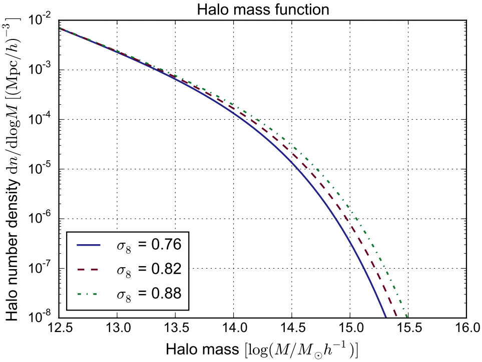
Unlensed sources



Weak lensing

Weak-lensing peak counts





- Local maxima of the projected mass
- Probe the mass function
- Non-Gaussian information

Dealing with selection function

Early studies

Count only the true clusters with high S/N

Recent studies

Include the selection effect into the model

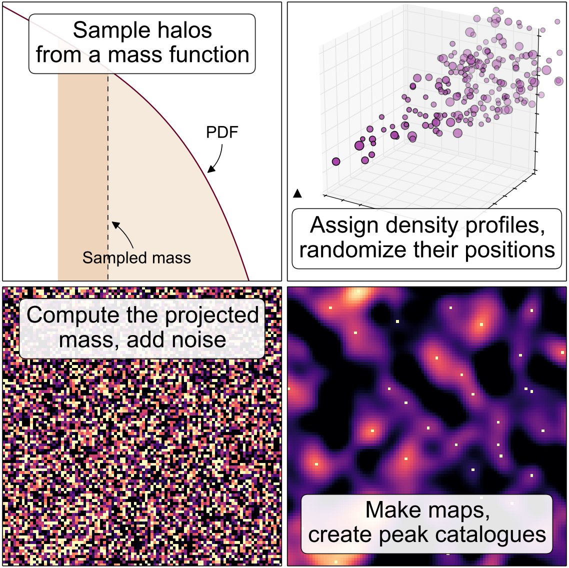
- Analytical formalism
- N-body simulations
- Fast stochastic model (this work)

Challenges

How to model weak-lensing peak counts properly and efficiently in realistic conditions?

What cosmological information can we extract from peaks?

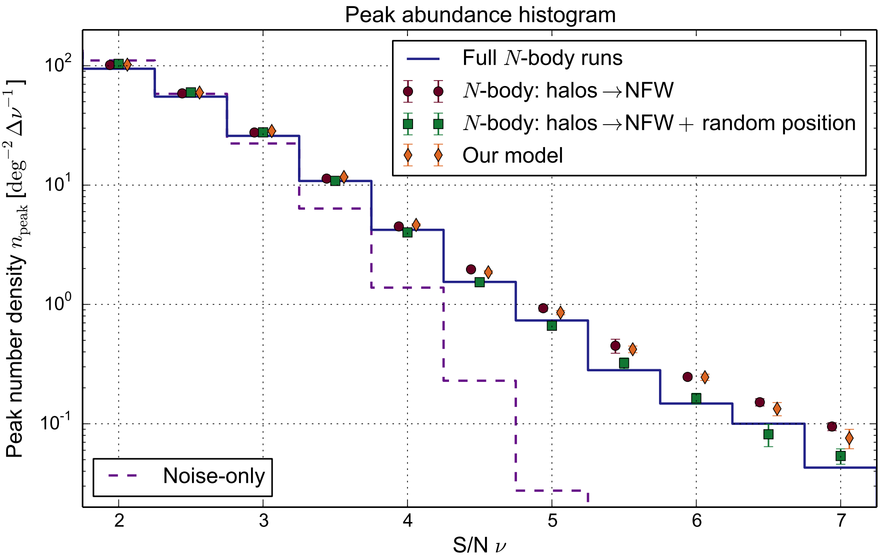
A new model to predict weak-lensing peak counts



Public code in C: Camelus@GitHub

See also Lin & Kilbinger (2015a)

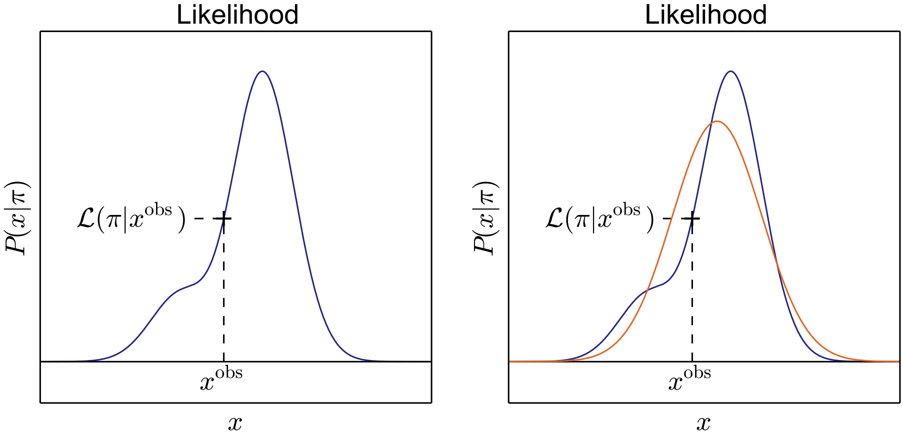
Validation



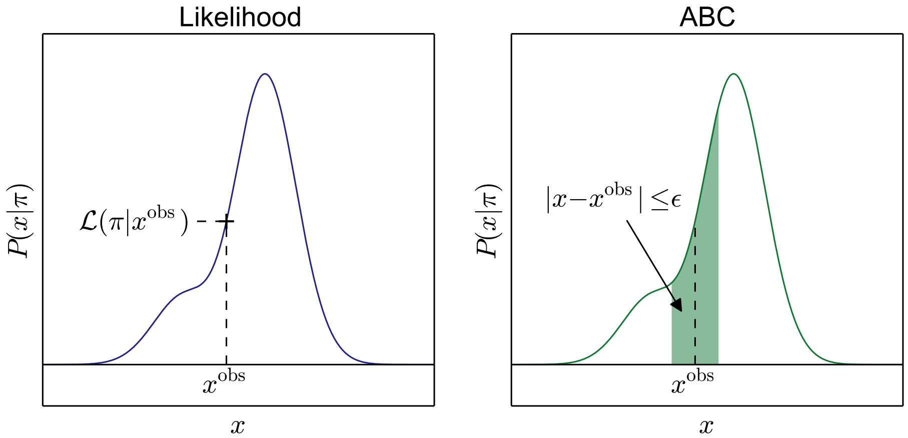
Lin & Kilbinger (2015a)

Likelihood

$$\mathcal{L}(\boldsymbol{\pi}|\boldsymbol{x}^{\mathrm{obs}}) \equiv P(\boldsymbol{x}^{\mathrm{obs}}|\boldsymbol{\pi})$$



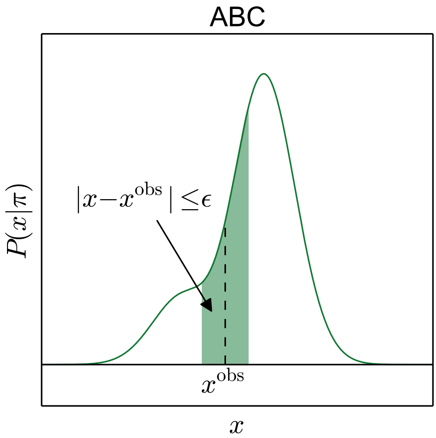
Approximate Bayesian computation



Approximate Bayesian computation

Requirements:

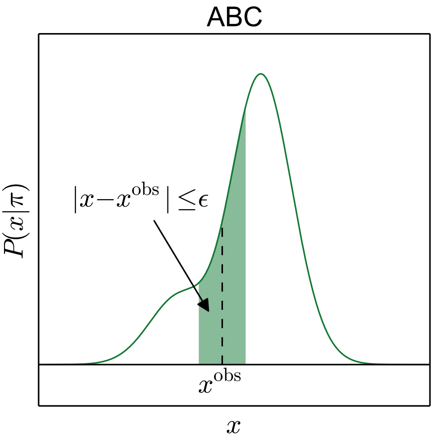
- Stochastic model $P(\,\cdot\,|\boldsymbol{\pi})$
- Distance $|\boldsymbol{x} - \boldsymbol{x}'|$
- Tolerance level ϵ



Approximate Bayesian computation

Accept-reject process:

- Draw a $\boldsymbol{\pi}$ from the prior $\mathcal{P}(\,\cdot\,)$
- Draw a $\boldsymbol{x}$ from the model $P(\,\cdot\,|\boldsymbol{\pi})$
- Accept $\boldsymbol{\pi}$ if $|\boldsymbol{x} - \boldsymbol{x}^{\mathrm{obs}}| \leq \epsilon$
- Reject otherwise

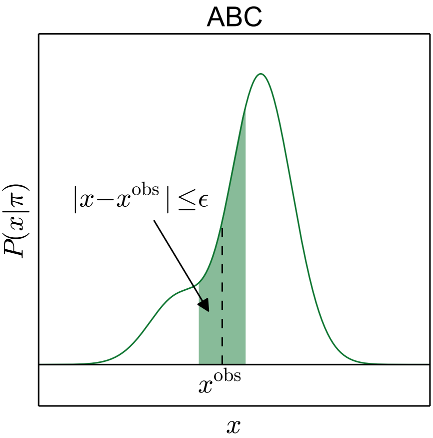


Approximate Bayesian computation

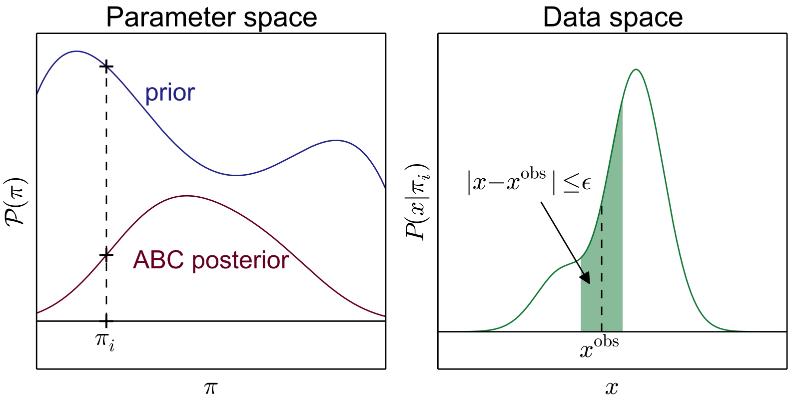
Accept-reject process:

- Draw a $\boldsymbol{\pi}$ from the prior $\mathcal{P}(\,\cdot\,)$
- Draw a $\boldsymbol{x}$ from the model $P(\,\cdot\,|\boldsymbol{\pi})$
- Accept $\boldsymbol{\pi}$ if $|\boldsymbol{x} - \boldsymbol{x}^{\mathrm{obs}}| \leq \epsilon$
- Reject otherwise

$\Rightarrow$ One-sample test



Approximate Bayesian computation



Distribution of accepted π $=$ prior $\times$ green areas

$\approx$ prior $\times\ 2\epsilon\ \times$ likelihood

$\propto$ posterior

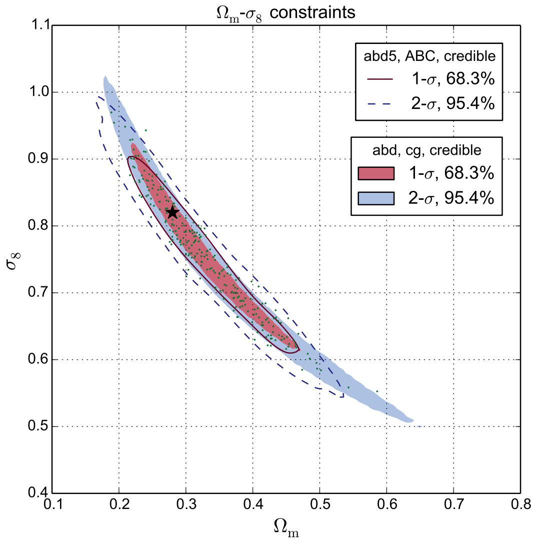
Combined with population Monte Carlo

Population Monte Carlo (PMC)

- Iterative solution for ϵ
- Set $\epsilon = +\infty$ for the first iteration
- Do ABC
- Update the prior based on results from the previous iteration
- Update ϵ based on results from the previous iteration
- Repeat until satifying a stop criterion

See also Lin & Kilbinger (2015b)

Comparison with the likelihood



Comparison by a toy model

Contours and dots: ABC

Colored regions: likelihood

Lin & Kilbinger (2015b)

Data from three surveys

Survey	Field size [deg^2]	Number of galaxies	Effective density [deg^{-2}]
CFHTLenS	126	6.1 M	10.74
KiDS DR1/2	75	2.4 M	5.33
DES SV	138	3.3 M	6.65

Various settings

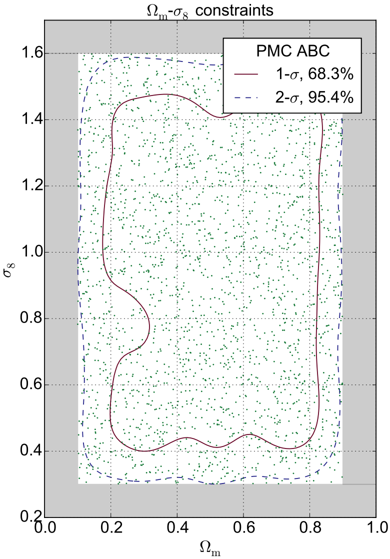
Model settings

- Compensated filter (suggested by Lin et al. 2016)
- Adaptive choice for pixel sizes and filtering scales
- No intrinsic alignment and baryons (not yet!)

ABC settings

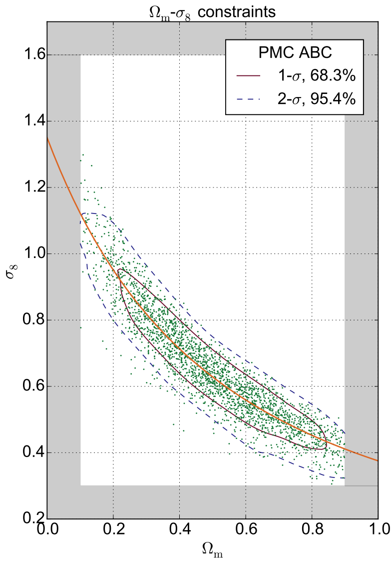
- Data vector (summary statistic): peak counts with S/N > 2.5 of all scales
- Distance: a χ^2-like normalized sum

Preliminary result



Lin & Kilbinger in prep.

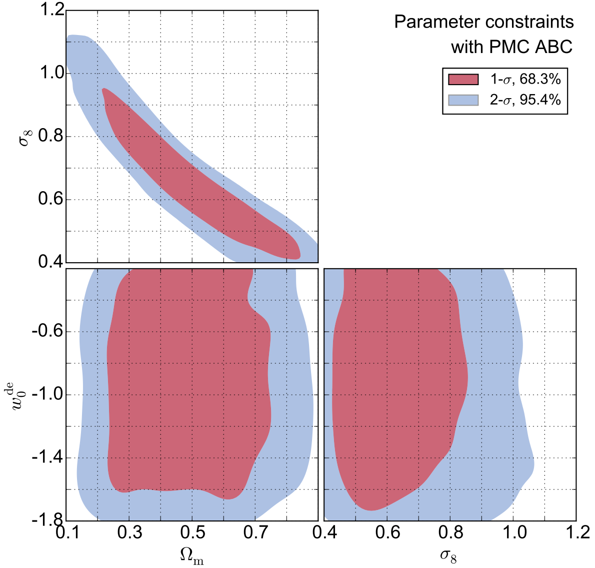
Preliminary result



Width: $\Delta\Sigma_8$ = 0.13

Area: FoM = 5.2

Preliminary result



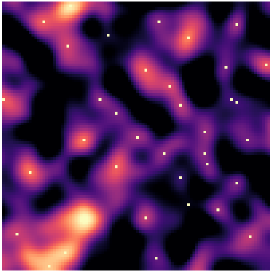
Lin & Kilbinger in prep.

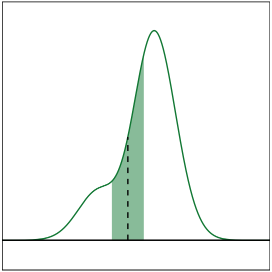
Ongoing improvements and perspectives

- S/N bin choice: less bias, more accuracy and precision
- Halo correlation
- Tomography
- Intrinsic alignment
- Baryonic effects

Summary

- A new model to predict WLPC
- Likelihood-free parameter inference: ABC
- Constraints with CFHTLenS, KiDS, DES





Collaborators:

Martin Kilbinger
Austin Peel (talk on Friday)
Sandrine Pires

References:

[1410.6955]
[1506.01076]
[1603.06773]
http://linc.tw

Backup slides

Advantages of our model

Fast

Only few seconds for creating a 25-deg^2 field, without MPI or GPU programming

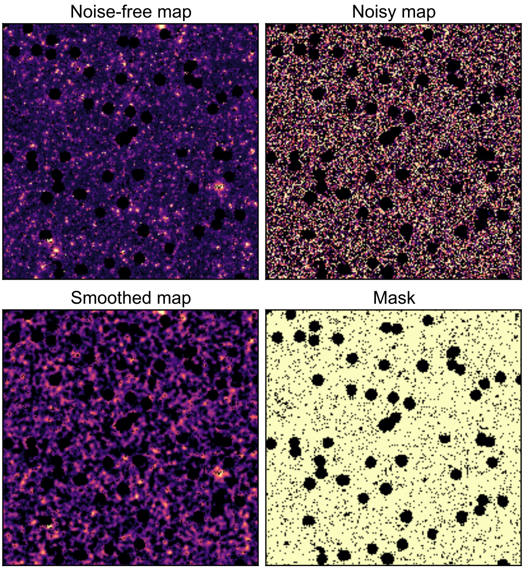
Flexible

Straightforward to include real-world effects (photo-z errors, masks, intrinsic alignment, baryonic effects, etc.)

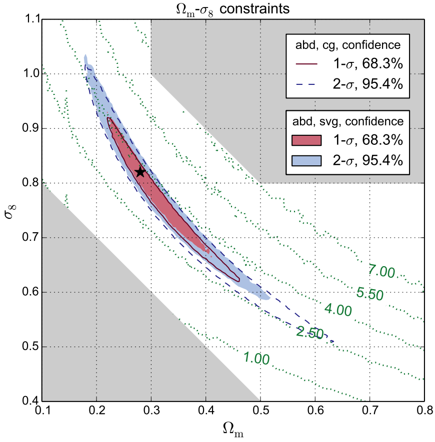
Full PDF information

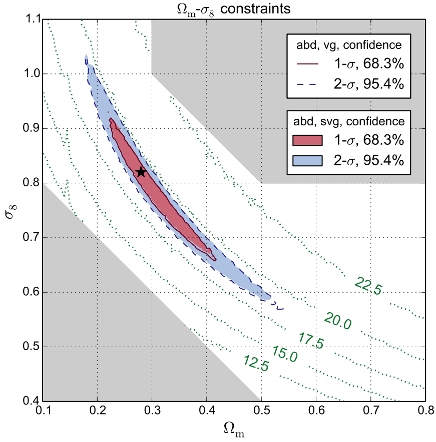
Allow more flexible constraint methods (varying covariances, copula, p-value, approximate Bayesian computation, etc.)

Map examples



Cosmology-dependent covariance

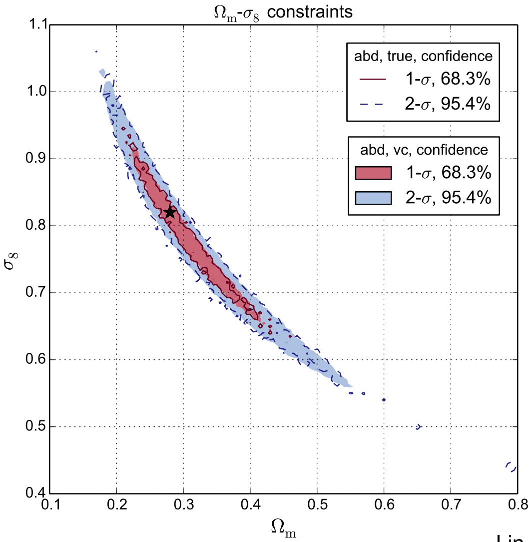




	cg	svg	vg
FoM	46	57	56

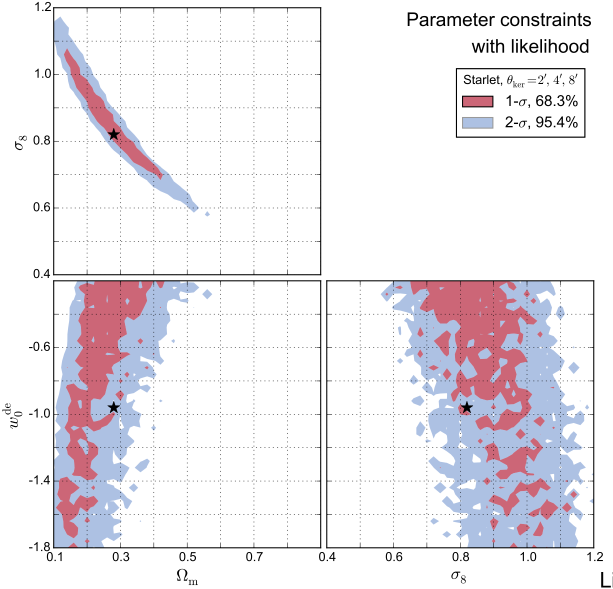
Lin & Kilbinger (2015b)

True likelihood



Lin & Kilbinger (2015b)

Degeneracy with $w_0^{\rm de}$



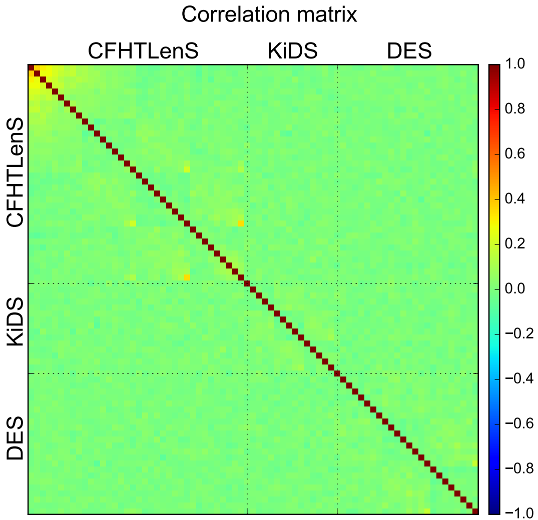
Parameter constraints with likelihood

Lin et al. (2016)

Technical detail

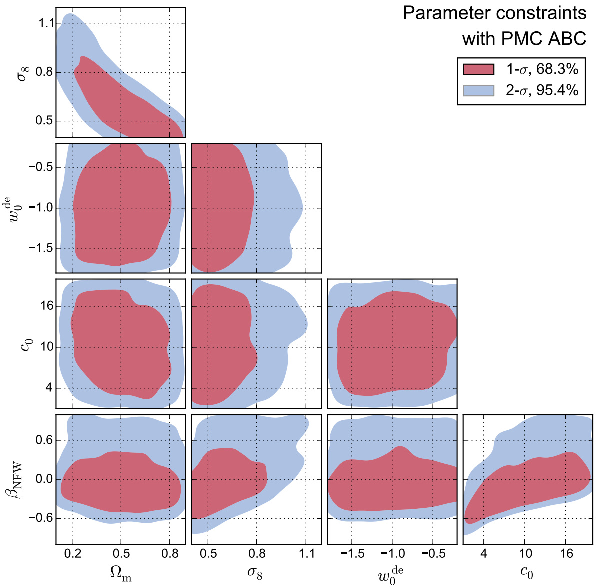
- Mass function from Jenkins et al. (2001)
- M-c relation from Takada & Jain (2002)
- Source redshift fitted from surveys
- Random source position, not catalogue
- Used raw galaxy densities, not effective
- Derive σ_ϵ from the emperical total variance
- Pixel size: $n_{\mathrm{gal}}\theta_{\mathrm{pix}}^2 \geq 7$
- Kaiser-Squires inversion
- Filtering with the “starlet” function
- Scale = 2, 4, 8 pixels
- Locally determined noise
- Dimension of x = 75 (= 36 + 15 + 24)

Covariance



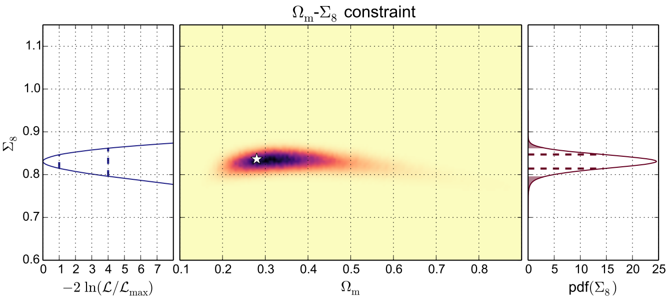
Lin & Kilbinger in prep.

Constrain concentration paramters



Lin & Kilbinger in prep.

Definition of Σ_8



Definition 1 $\Sigma_8 = \sigma_8(\Omega_m/0.27)^\alpha$

Σ_8 = 0.825 $\Delta\Sigma_8$ = 0.16 α = 0.48

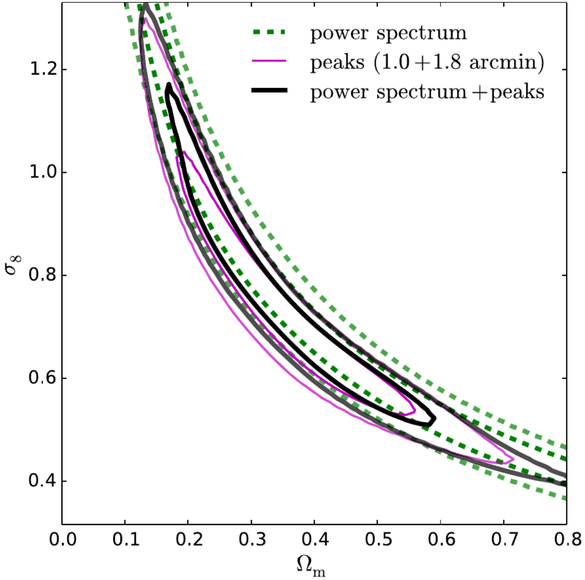
Definition 2 $\Sigma_8 = \left(\frac{\Omega_m+\beta}{1-\alpha}\right)^{1-\alpha}\left(\frac{\sigma_8}{\alpha}\right)^{\alpha}$

Σ_8 = 1.935 $\Delta\Sigma_8$ = 0.13 α = 0.38 β = 0.82

PMC ABC algorithm

set $t = 0$
for $i = 1$ to Q **do**
  generate $\boldsymbol{\theta}_i^{(0)}$ from $\mathcal{P}(\,\cdot\,)$ and $\boldsymbol{x}$ from $P\left(\,\cdot\,|\boldsymbol{\theta}_i^{(0)}\right)$
  set $\delta_i^{(0)} = D\left(\boldsymbol{x}, \boldsymbol{x}^{\mathrm{obs}}\right)$ and $w_i^{(0)} = 1/Q$
end for
set $\epsilon^{(1)} = \mathrm{median}\left(\delta_i^{(0)}\right)$ and $\boldsymbol{C}^{(0)} = \mathrm{cov}\left(\boldsymbol{\pi}_i^{(0)}, w_i^{(0)}\right)$
while success rate $\geq r_{\mathrm{stop}}$ **do**
  $t \leftarrow t + 1$
  for $i = 1$ to Q **do**
    repeat
      generate j from $\{1, \ldots, Q\}$ with weights $\left\{w_1^{(t-1)}, \ldots, w_Q^{(t-1)}\right\}$
      generate $\boldsymbol{\pi}_i^{(t)}$ from $\mathcal{N}\left(\boldsymbol{\pi}_j^{(t-1)}, \boldsymbol{C}^{(t-1)}\right)$ and $\boldsymbol{x}$ from $P\left(\,\cdot\,|\boldsymbol{\pi}_i^{(t)}\right)$
      set $\delta_i^{(t)} = D\left(\boldsymbol{x}, \boldsymbol{x}^{\mathrm{obs}}\right)$
    until $\delta_i^{(t)} \leq \epsilon^{(t)}$
    set $w_i^{(t)} \propto \mathcal{P}\left(\boldsymbol{\pi}_i^{(t)}\right) \Big/ \sum_{j=1}^{Q} w_j^{(t-1)} K\left(\boldsymbol{\pi}_i^{(t)} - \boldsymbol{\pi}_j^{(t-1)}, \boldsymbol{C}^{(t-1)}\right)$
  end for
  set $\epsilon^{(t+1)} = \mathrm{median}\left(\delta_i^{(t)}\right)$ and $\boldsymbol{C}^{(t)} = \mathrm{cov}\left(\boldsymbol{\pi}_i^{(t)}, w_i^{(t)}\right)$
end while

Peaks v.s. power spectrum



Taken from Liu J. et al. (2015)

Public code

Fast weak-lensing peak counts modelling in C
with PMC ABC

CAMELUS

Camelus@GitHub